NONEQUILIBRIUM HYPERSONIC FLOW OVER THE LATERAL SURFACE OF BLUNT BODIES

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1. This paper investigates viscous nonequilibrium flow of a gas along the lateral surface of blunt cones moving in air with high hypersonic speed. It is assumed that there is no electric current in the flow region, nor external electric or magnetic fields. The influence of radiation on the flow field is neglected. A nonpermeable wall is considered. The flow along the body is taken to be laminar.

In this formulation the problem of viscous nonequilibrium flow over long blunt bodies was considered in [1-3], where the calculated data are very limited and refer only to a few special flow cases. The present paper investigates the influence of conditions in the oncoming flow, the nose radius and the cone semiopening angle for a cone in air on the dynamic and thermal characteristics of the flow field, and also on the electron density.

The study is conducted by dividing the flow field into a boundary layer and an inviscid region. According to the results of [4-6], this approach to the solution of the problem is valid for Reynolds numbers, based on the blunting radius and the conditions at infinity, of $Re_\infty \geqslant 3\cdot 10^3$. As was shown in [5], the region of application of this approach is practically coincident with the region of application of the computational method based on the viscous shock-layer equations, used for calculations in [2]. However, the approach used in the present paper is more universal, since it allows one to obtain results even in the case of large Reynolds number, in particular, for turbulent flow near the wall.

Computation of the flow field in the "inviscid" formulation is carried out by the stream-tube method [7].

In the boundary-layer calculations, to simplify the problem it is assumed that only concentration diffusion occurs, which can be described using binary diffusion coefficients and constant values of Schmidt number. Judging from the data of [6], where the importance of calculating multicomponent diffusion was investigated in flow of hypersonic air over a body, this approximation should not appreciably affect the results.

Flow of a mixture of gases in the laminar boundary layer on a body with axial symmetry is described under the above assumptions by a system of equations of the type

$$\frac{\partial \rho v_s r(s)}{\partial s}+\frac{\partial \rho v_y r(s)}{\partial y}=0,\ \rho v_s\frac{\partial v_s}{\partial s}+\rho v_y\frac{dv_s}{dy}=-\frac{dp}{ds}+\frac{\partial}{\partial y}\left(\mu\frac{\partial v_s}{\partial y}\right),$$
$$\rho v_s\frac{\partial H}{\partial s}+\rho v_y\frac{\partial H}{\partial y}=\frac{\partial}{\partial y}\left\{\frac{\mu}{\Pr}\frac{\partial}{\partial y}\left[H+(\Pr-1)\frac{v_s^2}{2}\right]\right\}+\frac{\partial}{\partial y}\left\{\mu\sum_{i=1}^{N}\left(\frac{1}{Sc_i}-\frac{1}{\Pr}\right)h_i\frac{\partial \xi_i}{\partial y}\right\},\tag{1.1}$$
$$\rho v_s\frac{\partial \xi_i}{\partial s}+\rho v_y\frac{\partial \xi_i}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\mu}{Sc_i}\frac{\partial \xi_i}{\partial y}\right)+W_i,$$
$$p=\rho RT\sum_{i=1}^{N}\xi_i/m_i,\ H\equiv\sum_{i=1}^{N}h_i\xi_i+v_s^2/2\quad(i=1,2,\ldots,N),$$

where (s, y) is the natural coordinate system; $r(s)$, local body radius; v_s, v_y, tangential and normal components of the mean-mass velocity; ρ, density; p, pressure; μ, dynamic viscosity; Sc_i, Pr, Schmidt and Prandtl numbers; R, universal gas constant; T, temperature; h_i, ξ_i, enthalpy and relative mass concentration of the i-th component; W_i, mass rate of formation of the i-th component as a result of chemical reactions; H, total enthalpy of the mixture; and N, number of components of the mixture.

The conditions for velocity and temperature on the body surface are described in the form

$$y=0,\ v_s=v_y=0,\ T=T_w.\tag{1.2}$$

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It is assumed that equilibrium values of the concentration of all components hold at the wall: $\xi_i = \xi_{iw}(T, p)$.

A characteristic for flow over blunt bodies is the presence of large transverse gradients of all the parameters near the body, due to the presence of a curved shock wave ahead of the body. This circumstance, as was shown in [8], should be taken into account in calculating the boundary layer (in the case of moderate Reynolds numbers). The conditions at the outer edge of the boundary layer, allowing for nonuniformity of the external flow, take the form

$$y = \delta,\ v_s = v_e(\psi_e),\ T = T_e(\psi_e),\ \xi_i = \xi_{ie}(\psi_e), \tag{1.3}$$

where ψ_e is the stream function;

$$\psi_e = \int_0^\delta \frac{\rho v_s r(s)\, dy}{\rho_\infty v_\infty r_0^2} \tag{1.4}$$

(r_0 is the radius of blunting). To determine δ we use the condition $y = \delta$, $\partial v_s/\partial y = 0$.

Thus, the conditions at the outer edge cannot be assigned beforehand, as is done in the usual formulation of a boundary-layer problem. To solve the problem we can use the method of successive approximations, by assigning a stream function in the zero-order approximation, $\psi_e^0(s)$, and integrating the system of boundary-layer equations (1.1) successively for ψ_e^k ($k = 0, 1, 2, \ldots$), calculated from Eq. (1.4). This cumbersome procedure is simplified if we can manage to assign the zero-order approximation $\psi_e^0(s)$. An investigation conducted in [9] has shown that for spherically blunted cones with semiopening angles $\alpha = 5$–$15°$ the quantity ψ_e can be calculated from the relation

$$\psi_e = \frac{A s_1^2 K^2}{\sqrt{\mathrm{Re}_\infty/\mathrm{M}_\infty}},\quad s_1 = \frac{s}{r_0}, \tag{1.5}$$

where M_∞ is the Mach number of the incident flow; the relation $K = K(s_1)$ is given graphically [9]. Our calculations have confirmed this relation for

$$A = 5.8 + 0.4/s_1. \tag{1.6}$$

In using Eqs. (1.5) and (1.6) it proves to be possible to confine ourselves to only the zero-order approximation of the above procedure, i.e., to integrate the system of equations (1.1) with boundary conditions (1.2) and (1.3) only once.

In practice the determination of conditions at the outer boundary reduces to finding, from the data in calculating the inviscid region of number $k(s_1)$, stream tubes satisfying the condition

$$\sum_{j=1}^{k-1} Q_j < \psi_e(s_1) \leqslant \sum_{j=1}^{k} Q_j,$$

where Q_j is the relative mass flow rate of gas through the j-th stream tube. The conditions at the outer boundary at the point s_1 are considered to coincide with the parameters of the k-th tube at the point s_1.

Integration of the system of equations (1.1) with boundary conditions (1.2) and (1.3) is accomplished by a multistrip integral method [10], using four strips.

It is assumed that the air mixture is a mixture of seven components (O_2, N_2, O, N, NO, NO^+, e) between which the following reactions occur:

$$O_2 + M \rightleftarrows 2O + M,\ N_2 + M \rightleftarrows 2N + M,$$
$$NO + M \rightleftarrows N + O + M,\ NO + O \rightleftarrows N + O_2,$$
$$N_2 + O \rightleftarrows NO + N,\ N_2 + O_2 \rightleftarrows 2NO,\ N + O \rightleftarrows NO^+ + e.$$

The rate constants for these reactions are given the same values as in [11].

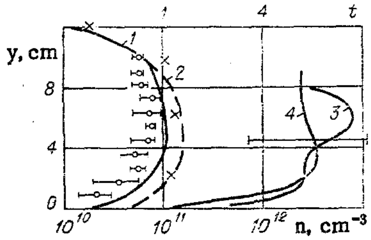

Fig. 1

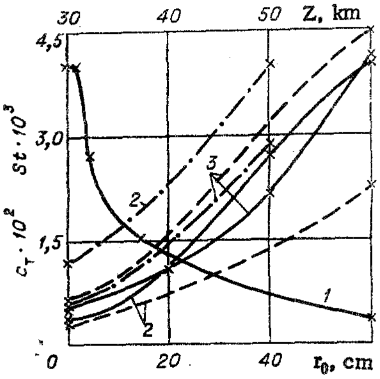

Fig. 2

2. In [12] test data were presented on ionization near the surface of a cone with blunting radius r_0 = 15 cm, $\alpha = 9°$, moving at height Z = 71 km with velocity v_∞ = 7.6 km/sec. The test data on the distribution of electron density n through the shock layer thickness y at the section s_1 = 8.8 are shown in Fig. 1 as circles with horizontal lines, showing the scatter of the test data. (We have rejected experimental points, from probes whose readings cannot be accurate, according to [13].) In order to compare with these data we carried out an appropriate computation with the above method. We assumed T_W = 1000°K, Pr = 0.7, Sc_i = 0.5 for the neutral components and Sc_i = 0.25 for the charged component. Curve 1 of Fig. 1 corresponds to the calculation using the electron recombination rate constants from [13], the crosses correspond to the data of [14], and curve 2 corresponds to the calculated data obtained in [2] in the viscous shock-layer approximation. The agreement is quite close, both with the test data, and with the calculated values obtained using a different computational method. It can be seen that better agreement with the test data is obtained when one used recombination rate constants from [13].

Curves 3 and 4 in Fig. 1 are the calculated temperature profiles at sections s_1 = 2.1 and 8.8, respectively. The horizontal line shows the scatter of test data on electron temperature $t = T/10^3$. These data were taken at a distance of 1.5–9.5 cm from the wall and were presented in [15]. It can be seen that the electron temperature does not differ appreciably from the heavy-particle temperature.

Figures 2–4 illustrate the influence of flight altitude and the geometrical parameters of the cone, moving with speed v_∞ = 7.4 km/sec, on the basic flow characteristics near the wall at the section $s = s_* = 1$ m. These calculations assume the above values of T_W, Pr and Sc_i.

Figure 2 shows values of the total friction drag coefficients of the cones

$$c_T = \frac{4}{\rho_\infty v_\infty^2 r_*^2} \int_0^{s_*} \tau_w r \cos\beta \, ds, \quad \tau_w \equiv \mu_w \left(\frac{\partial v_s}{\partial y}\right)_w,$$

where β is the angle of inclination of the tangent to the body contour; $r_* = r(s_*)$, and the Stanton number is

$$\mathrm{St} = \frac{q_w}{\rho_\infty v_\infty H_\infty (1 - H_w/H_\infty)}, \quad q_w = -\mu_w \left(\frac{c_p}{\Pr}\frac{\partial T}{\partial y} - \sum_{i=1}^{N} \frac{h_i}{\mathrm{Sc}_i}\frac{\partial \xi_i}{\partial y}\right)_w,$$

$$c_p = \sum_{i=1}^{N} \xi_i \frac{dh_i}{dT}.$$

Curve 1 shows c_T as a function of the body blunting radius r_0 for the case z = 50, $\alpha = 10°$. Curves 2 and 3 show c_T and St as a function of the flight height z. Here and in Fig. 3 the solid lines refer to the case r_0 = 4 cm, $\alpha = 10°$, the broken lines to the case r_0 = 4 cm, $\alpha = 6°$, and the dot–dash lines to a sharp cone with $\alpha = 10°$ (the corresponding calculations for sharp cones were presented in [10]). It should be borne in mind that all the curves were constructed on the basis of a relatively small number of calculated cases. All the calculated points in Figs. 1–4 are marked by crosses along the curves.

Figure 3 shows the variation with flight altitude of the boundary layer thickness δ (cm), the maximum values of the temperature t_1, and the electron density n_1 (cm^{-3}) near the wall. Figure 4 shows the same parameters as a function of body blunting radius for the case Z = 50, $\alpha = 10°$.

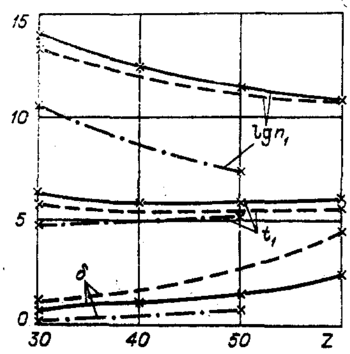

Fig. 3

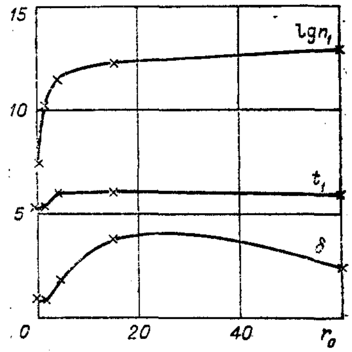

Fig. 4

It is interesting to note that in spite of the fact that the pressure on the lateral surface of the cone in the case $\alpha = 6°$ is less, by a factor of 2-3, than for the case $\alpha = 10°$, and somewhat below the temperature (Fig. 3), the electron density at $Z = 60$ for $\alpha = 6°$ proves to be practically the same as for $\alpha = 10°$. As analysis of the calculated data shows, this is due to the fact that at lower pressure levels there is more intense freezing of the chemical processes, the result being a considerably higher level of concentration of atoms of oxygen and nitrogen along the lateral surface of the body. The latter leads to an increase in the rate of formation of electrons resulting from the dissociative recombination reaction. This effect is seen less strongly at lower altitudes.

We note one further important numerical result: As can be seen from the data shown in Figs. 2 and 4, the dynamic and thermal characteristics of the boundary layer on a cone of blunting radius $r_0 = 1$ cm are practically the same (at the section examined) as for a sharp cone. However, the ionization for the blunt cone is greater by almost three orders of magnitude.

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